The Diffuse View Factors between Enclosed Surfaces in a Rotary Kiln Incinerator

Y. Y. CHEN
Department of Chemical Engineering, Yuan-Ze Institute of Technology, Taoyuan, Taiwan 32026, R.O.C.

R. H. CHANG
Energy and Resources Laboratories, Industrial Technology Research Institute, Hsinchu, Taiwan 31015, R.O.C.

D. J. LEE
Department of Chemical Engineering, National Taiwan University, Taipei, Taiwan 10617, R.O.C.

Key Words: Diffuse View Factor, Rotary Kiln Incinerator, Working Equations.

INTRODUCTION

In many high temperature industrial applications, the radiation heat transfer plays a very important role in the heat/mass transfer processes, as in the rotary kiln process (Watkinson and Brimacombe, 1978; Tscheng and Watkinson, 1979; Gorog, et al., 1982; Barr, et al., 1989a, 1989b; Chang, 1990). To estimate the contribution of radiation heat transfer rate between nonisothermal enclosures in such processes, the view factors are the basic information needed in calculation. The diffuse view factors for many simple geometry had been evaluated in literature and were summarized in several books (Ozisik, 1973; Sparrow and Cess, 1978; Siegel and Howell, 1981). However, all the diffuse view factors needed in rotary kiln processes were still lack to the authors' best knowledge.

The basic geometry of a rotary kiln was a circular hollow tube partially filled with a solid bed and was bounded by two side walls. Since the temperature did not distribute uniformly along the solid bed and the tube wall, and since the side walls might also have large contribution on the radiation heat transfer, the diffuse view factors between the solid bed, the refractory wall, and the side walls were needed.

In this note, the diffuse view factors between various enclosed surfaces in a rotary kiln were evaluated by contour integration followed by numerical integration. The results were correlated into several working equations which were simple in form and could be applied easily in further analysis.

ANALYSIS

In the following analysis, the view factors between the surfaces shown in Fig. 1a to 1e would be referred to as Case A to Case E, respectively. \( F_{dP_1-A_1} \), the diffuse view factor between an elemental surface \( dP \), and the surface \( A_1 \), shown in Fig. 2(a), and \( F_{dA_1-A_2} \), the diffuse view factor between elemental surface \( dA \), and surface \( A_1 \), shown in Fig. 2(b), will be needed in further calculations. Therefore, the analytical solutions of these two factors were derived in Appendix I.

In this work, the fill angle \( 2\theta' \) was taken as a constant along the kiln. It was noted that the view factor between two surfaces will remain unchanged if the surfaces exchange according to their axial positions. Therefore, only the conditions when the position vector between two enclosed surfaces was positive would be discussed.

1. Case A

Consider the diffuse view factor between \( dP \), and \( A_1 \) in Fig. 1(a), it was clear that the value could be evaluated by direct integration of as:

\[
2R \sin \theta' dz_1 F_{dP_1-A_1} = \int_{dP_1} F_{dA_1-A_1} dy_1 dz_1 \tag{1}
\]

Therefore,
Fig. 1(a). Diffuse view factor between segment \(dP_1\) and surface \(A_p\).

Fig. 1(b). Diffuse view factor between segment \(dP_1\) and band \(dB_2\).

Fig. 1(c). Diffuse view factor between surfaces \(A_3\) and \(A_4\).

Fig. 1(d). Diffuse view factor between band \(dB_3\) and surface \(A_4\).

Fig. 1(e). Diffuse view factor between band \(dB_3\) and band \(dB_4\).

**Fig. 2.** Coordinate system employed in this work.

\[
F_{dp_{1}}_{A_p} = \frac{1}{2R \sin \theta} \int _{-R \sin \theta}^{R \sin \theta} F_{dA_1-A_p} dy_1
\]

\[
= \frac{1}{2 \sin \theta} \left[ \int _{-R \sin \theta}^{+R \sin \theta} F_{dA_1-A_p} dy_1 \right] \tag{2}
\]

where \(y^+_1 = y_1/R\).

Since there existed no analytical solution of Eq. (2), numerical integration was needed.

2. Case B

The view factor between \(dP_1\), to the band surface \(dB_2\), shown in Fig. 1(b) could be obtained as:

\[
dF_{dp_{1}}_{dB_2} = F_{dp_{1}}_{A_p} \int _{-R \sin \theta}^{R \sin \theta} F_{dA_1-A_p} dy_1
\]

\[
= \frac{1}{2R \sin \theta} \int _{-R \sin \theta}^{R \sin \theta} (F_{dA_1-A_p})_\delta (dZ_4) \tag{3}
\]

Substitute Eq. (2) into Eq. (3), the result was:

\[
dF_{dp_{1}}_{dB_2} \frac{1}{2R \sin \theta} \frac{d}{dZ_4} \left[ \int _{-R \sin \theta}^{R \sin \theta} (F_{dA_1-A_p} dy_1) dZ_4 \right] \tag{4}
\]

The following relation held:

\[
\frac{d}{dZ_4} \int _{-R \sin \theta}^{R \sin \theta} (F_{dA_1-A_p} dy_1) = \int _{-R \sin \theta}^{R \sin \theta} (\frac{\partial}{\partial Z_4} F_{dA_1-A_p}) dy_1 \tag{5}
\]
Thus, Eq. (4) became:

$$\frac{dF_{dA_{-dA}}}{d\theta} = \frac{1}{2 \sin \theta} \int_{-\theta}^{\theta} \frac{\partial}{\partial Z} F_{dA_{-dA}} dy \frac{dZ}{d\theta}$$ (6)

The differentiation of \( F_{dA_{-dA}} \) with respect to \( Z^* \) in Eq. (6) could be shown analytically as follows:

$$Y_{e} = \frac{c}{b^2 + c^2} \frac{4Z^* \sin \theta}{(a + b \cos \theta - c \sin \theta)(a + b \cos \theta + c \sin \theta)}$$

$$Y_{e} = -\frac{4bZ^*}{(a^2 - b^2 - c^2)^{1/2}}$$

$$Y_{e} = -2Z^* \tan \left( \frac{\theta}{2} \right) \left( a^2 - b^2 - c^2 \right) - 2aZ^* \left[ (-a - b) \tan \left( \frac{\theta}{2} \right) + c \right]$$

$$Y_{e} = 2Z^* \tan \left( \frac{\theta}{2} \right) \left( a^2 - b^2 - c^2 \right) - 2aZ^* \left[ (a - b) \tan \left( \frac{\theta}{2} \right) + c \right]$$

$$Y_{e} = (b_i - a_i)Z^* \cos \theta$$

$$Y_{e} = \frac{(b_i - a_i)Z^* \left[ \sin \theta \theta \right] + c}{(a_i - b_i - c_i)^{1/2}}$$

$$Y_{e} = \frac{(b_i - a_i)Z^* \left[ - \sin \theta \theta \right] + c}{(a_i - b_i - c_i)^{1/2}}$$

Numerical integration was also needed to evaluate the integration term in Eq. (6).

3. Case C

The diffuse view factor between \( dA \) and \( A_1 \) in Fig. 1(c) was shown in Eq. (A-8). Therefore, the diffuse view factor from \( A_1 \) to \( A_2 \) could be found from surface integration as:

$$F_{dA_{-dA}} = \frac{1}{A_2} \int_{A_1} F_{dA_{-dA}} dA$$

(9)

Numerical double integration was required in this case.

4. Case D

From point of conservation of energy, the following equation held for the view factors shown in Fig. (1d):

$$\frac{\partial}{\partial Z} F_{dA_{-dA}} = -\frac{1}{2\pi} \left[ Y_{e} + Y_{e} + Y_{e} + Y_{e} \right]$$

(11)

From Eqs. (2), (9), and (10), the following result could be obtained by applying Leibniz's rule:

$$F_{dA_{-dA}} + F_{dA_{-dA}} = \frac{\partial}{\partial Z} F_{dA_{-dA}}$$

(10)

In Eq. (11), the last term could be found from Eq. (2) and the differentiation term could be shown analytically as

$$\frac{\partial}{\partial Z} F_{dA_{-dA}} = -\frac{1}{2\pi} \left[ Z_{e} + Z_{e} + Z_{e} \right] + Z_{e} \frac{1}{4\pi} \frac{d}{d\theta} \left[ Z_{e} (Z_{e} + Z_{e}) + Z_{e} (Z_{e} + Z_{e}) \right]$$

(12)
Numerical integration was also needed to evaluate the double integration in Eq. (11).

5. Case E

From the law of the conservation of energy, the following equation held:

$$dF_{\theta_{1}-\theta_{2}} + dF_{\theta_{2}-\theta_{3}} = \frac{\partial}{\partial z} (F_{\theta_{1}-\theta_{3}}) dZ, \quad (14)$$

Substitute Eq. (11) into Eq. (14) we can get

$$dF_{\theta_{1}-\theta_{2}} = \frac{\partial}{\partial z} (F_{\theta_{1}-\theta_{3}}) dZ - dF_{\theta_{1}-\theta_{3}}$$

To evaluate Eq. (15), double integration of the second order differentiation of $F_{\theta_{1}-\theta_{3}}$ was needed. But it was found in the latter calculations that the direct integration of the second order differentiation form was numerically undesirable, since the integration became very stiff when $|Z_{\theta}|$ approached zero. Therefore, the first order differentiation form Eq. (12) was integrated first, and then numerical differentiation was applied to evaluate the required value in Eq. (15).

**NUMERICAL METHODS**

The view factors $F_{\theta_{1}-\theta_{2}}$, $dF_{\theta_{1}-\theta_{2}}$, $F_{\theta_{2}-\theta_{3}}$, $dF_{\theta_{2}-\theta_{3}}$, and $dF_{\theta_{3}-\theta_{4}}$ would be obtained if Eqs. (2), (9), (11), and (15) could be evaluated. Since no analytical solutions of these equations were available, numerical solutions were needed.

To perform the single integration in this work (Eqs. (2) and (6)), Gear’s method was directly applied. The relative errors were all controlled to be under $10^{-4}$. For the double integration calculations (Eqs. (9), (11) and (15)), the inner shell of integration (the $r$ domain) were evaluated by Gear’s method, and the outer shell of integration (the $\theta$ domain) were divided into $2^n$ panels and each panel was calculated by six-point Gaussian quadrature. To check the convergence of the integrations, the results from $2^n$ panels were compared with the integral values form the $2^{n+1}$ panels. If the result was not converged, each panel is divided into two sub panels, and the procedure was repeated. The tolerance used in double integrations was $10^{-4}$. Usually, the $N$ value needed for satisfactory integration was about 6 to 7.

To evaluate Eq. (15), central difference scheme was used in numerical differentiation calculations.
RESULTS AND DISCUSSIONS

General

The view factors evaluated from Eqs. (2), (6), (9), (11) and (15) with $\theta^0$ as the parameter when shown in Figs. 3 to 7. It was clear that the view factors all decreased as $Z^*_0$ increased and would approach to zero when $Z^*_0 \rightarrow \infty$. It was also clear that the view factors would decrease to below 0.05 when $|Z^*_0|$ was larger than about 3.0. Therefore, for a long rotary kiln, the enclosed surfaces which were three radius away from the surface could be neglected in practice. The effects of $\theta^0$ on $F_{A_1-A_2}$, $F_{A_2-A_3}$, and $F_{A_3-A_4}$ were minor when compared with the effects of $Z^*_0$, which suggested that these view factors could also be used in the applications whose $\theta^0$ changed continuously along the tube.

Fig. 3. Diffuse view factor $F_{A_1-A_2}$ vs. $Z^*_0$. The correlation equation (Eq. (21)) was shown as solid lines for comparisons.

Fig. 5. Diffuse view factor $F_{A_1-A_2}$ vs. $Z^*_0$. The correlation equation (Eq. (23)) was shown as solid lines for comparisons.

Fig. 4. Diffuse view factor $dF_{A_1-A_2}$ vs. $Z^*_0$. The correlation equation (Eq. (22)) was shown as solid lines for comparisons.

Fig. 6. Diffuse view factor $F_{A_3-A_4}$ vs. $Z^*_0$. The correlation equation (Eq. (24)) was shown as solid lines for comparisons.
Though there existed no analytical and/or numerical results in literature with which our calculation results could be compared, several physical interpretations were used to check the accuracy of the current work. First, when the segment $dP_i$ and surface $A_i$ in Fig. 1(a) approached to each other, the view factor from $dP_i$ to $A_i$ should approach 0.5, i.e.,

$$\lim_{Z_0 \to 0} F_{dP_i-A_i} = 0.5$$

(16)

Similarly,

$$\lim_{Z_0 \to 0} F_{A_3-A_4} = 0.5$$

(17)

and

$$\lim_{Z_0 \to 0} F_{A_2-A_3} = 1.0$$

(18)

These properties were clearly shown in Figs. 3, 5, and 6.

Second, when $\theta = 0$, $F_{dA_3-A_4}$ became the view factor from elemental surface to a circular disk which could be found in Sparrow and Cess (1980) and $dF_{dA_3-A_4}$ should be the same as the view factor between two elemental coaxial circular bands which was also listed in Ozisik (1973). Figure 8 showed the numerical results of Eqs. (9) and (15) ($\theta = 0.001$) and the analytical solutions (the lines). It was clear that the numerical results coincided with the analytical solutions.

Third, when a segment $dP_i$ was placed at the edge of a semi-infinite tube (as shown in Fig. 9), half of the radiative energy leaving $dP_i$ would be captured by the tube wall, i.e.,

$$I_i = \int_{z_0^+}^{z_0^+} dF_{dP_i-dA_i} = 0.5$$

(19)

For the same reason, the following relation also held:

$$I_i = \int_{z_0^+}^{z_0^+} dF_{dA_3-dA_4} + dF_{dA_3-dP_i} = 0.5$$

(20)

The calculated view factors were then integrated by Simpson's rule and the results were shown in Figs. 10(a) and 10(b). It was clear that Eqs. (19) and (20) held with errors less than 1% when $|Z_0^+|$ was larger than about 6.0.
From the above discussions, it was believed that the current results were accurate enough for most applications.

**Working equations**

For the sake of convenience for further usage, the view factors evaluated numerically were correlated as simple functional form as the working equations. The results for Case A to Case E were listed as follows.

1. **Case A**

   \[
   F_{d_{1_{1}}-\lambda_{1}} = \sum_{i=1}^{4} \alpha_{i}|Z_{+}|^{i} 
   \]
   (21a)

   where

   \[
   \alpha_{i} = \sum_{j=0}^{i} \alpha|^{j}
   \]
   (21b)

2. **Case B**

   \[
   dF_{d_{1_{1}}-\lambda_{2}} = \sum_{i=0}^{5} \beta_{i}|Z_{+}|^{i}dZ_{+}^{i}
   \]
   (22a)

   where

   \[
   \beta_{i} = \sum_{j=0}^{i} \beta|^{j}
   \]
   (22b)

3. **Case C**

   \[
   F_{d_{2_{1}}-\lambda_{1}} = F_{d}(r_{2} + 7, \theta + 7, \theta^{+})
   \]
   (23a)

   where

   \[
   F_{d} = \sum_{i=0}^{6} \alpha_{i}|Z_{+}|^{i}
   \]
   (23b)

4. **Case D**

   \[
   F_{d_{3_{1}}-\lambda_{1}} = \sum_{i=0}^{4} \delta_{i}|Z_{+}|^{i}
   \]
   (24a)

   where

   \[
   \delta_{i} = \sum_{j=0}^{i} \delta|^{j}
   \]
   (24b)

The coefficients \(\alpha_{i}, \beta_{i}, \gamma_{i}, \sigma_{i}, \delta_{i}\) were all listed in Appendix II.

5. **Case E**

   \[
   dF_{d_{1_{1}}-\lambda_{1}} = N \exp (N|Z_{+}|)dZ_{+}^{i}
   \]
   (25)

   where

   \[
   M = -0.20310^{2} + 0.5529 \]
   (25b)

   \[
   N = -0.21716(1 - \cos \theta)^{2}
   \]
   (25c)

The working equations Eqs. (21) to (25) were also shown as solid lines in Figs. 3 to 7. It was shown that for most cases, the averaged relative errors would be within 2%. These equations were valid within \(|Z_{+}| \leq 3\). When \(|Z_{+}|\) was larger than 3, the view factors could be taken as zero in practice.
CONCLUSIONS

The diffuse view factors between enclosed surfaces in a rotary kiln were evaluated by contour integration followed by numerical integration. The accuracy of the calculations were checked by several physical interpretations. The results were correlated into several working equations which were simple in form and could be applied in further analysis.

ACKNOWLEDGEMENTS

This work was supported by Energy and Resources Laboratories, Industrial Technology Research Institute, Hsinchu, Taiwan, R.O.C.

NOMENCLATURE

\( A \)  
area, (m²)

\( F \)  
diffuse view factor, (-)

\( I_1, I_2 \)  
integrations defined in Eqs. (19) and (20)

\( R \)  
kiln radius, (m)

\( r \)  
distance, (m)

\( y_1, y_2 \)  
positions at \( y \) direction, (m)

\( z_1, z_2, z_1, z_2 \)  
positions at \( z \) direction, (m)

Greek Letter

\( \theta \)  
half of the fill angle, (rad)

REFERENCES


APPENDIX I

Consider two elemental surfaces \( dA_1 \) and \( dA_2 \) with uniform temperatures and the normal vector were \( \hat{n}_1 \) and \( \hat{n}_2 \) respectively. The diffuse view factor between surfaces \( dA_1 \) and \( dA_2 \), was defined as:

\[
dF_{dA_1-dA_2} = \frac{\cos \theta_1 \cos \theta_2}{\pi r^2}
\]

The view factor from \( dA_1 \) to a finite surface \( A_s \) can be determined by integrating \( dF_{dA_1-dA_2} \) over surface \( A_s \), as

\[
F_{dA_1-dA_2} = \int_{A_s} F_{dA_1-dA_2} dA
\]

The evaluation of view factors in equations (A-1) and (A-2) can be performed by applying the stokes theorem to reduce the surface integral to contour integral as in Ozisik (1973).

In the following derivations, the diffuse view factors \( F_{dA_1-A_s} \) and \( F_{dA_2-A_s} \) needed in further analysis were evaluated analytically as follows.

1. \( F_{dA_1-A_s} \) in Fig. 2(a)

For the two surfaces \( dA_1 \) and \( A_s \) shown in Fig. 2(a), the diffuse view factor could be represented as:

\[
F_{dA_1-A_s} = \frac{-1}{2\pi} \int_{A_s} \left( \frac{dy_2}{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \right)
\]

Change the coordinate into cylindrical coordinate and integrated analytically, the result was

\[
F_{dA_1-A_s} = \frac{-Z^2}{2\pi} \left[ Y_s^2 + Y_s^2 + Y_s(Y_s + Y_s) \right] + Y_s(Y_s + Y_s)
\]

where

\[
r^+ = \frac{r}{R}
\]

\[
Z^2 = \frac{z}{R}
\]

\[
Y_s = \frac{c}{b^2 + c^2} \ln \left( \frac{a + b \cos \theta + c \sin \theta}{a + b \cos \theta + c \sin \theta} \right)
\]
Y. Y. Chen, R.H. Chang and D.J. Lee: The Diffuse View Factors Between Enclosed Surfaces in a Rotary Kiln Incinerator

\[ Y_d = \frac{2b(\pi - \theta^e)}{b^2 + c^2} \]  
(A-5d)

\[ Z_d = \theta^e - \pi \]  
(A-8a)

\[ Z_1 = \frac{r_1^e + Z_1^e - 1}{\sqrt{a^2 - b^2 - c^2}} \]  
(A-8b)

\[ Z_2 = \tan^i \left[ \frac{-(a-b) \tan \left( \frac{\theta^e}{2} \right) + c}{\sqrt{a^2 - b^2 - c^2}} \right] \]  
(A-8c)

\[ Z_3 = \tan^i \left[ \frac{(a-b) \tan \left( \frac{\theta^e}{2} \right) + c}{\sqrt{a^2 - b^2 - c^2}} \right] \]  
(A-8d)

\[ Z_4 = \tan^i \left[ \frac{(\sin 2\theta^e) + c_1}{\sqrt{a^2 - b^2 - c^2}} \right] \]  
(A-8e)

\[ Z_5 = \tan^i \left[ \frac{(\sin 2\theta^e) + c_1}{\sqrt{a^2 - b^2 - c^2}} \right] \]  
(A-8f)

\[ Z_6 = \frac{2}{\sqrt{a^2 - b^2 - c^2}} \]  
(A-8g)

\[ a_i = \cos \theta^e + b_i \]  
(A-8h)

\[ b_i = \frac{1}{2} (r_1^e + Z_1^e - 2 \cos \theta^e \cos \alpha_i r_1^e) \]  
(A-8i)

\[ c_i = -\cos \theta^e \sin \theta^e r_i^e \]  
(A-8j)

\[ d_i = -\cos \theta^e \cos \theta^e r_i^e \]  
(A-8k)

The coefficients of Eqs. (21) to (25) were shown in this Appendix.

**APPENDIX II**

The coefficients of Eqs. (21) to (25) were shown in this Appendix.

1. Case A

\[ a_i^A = 0.5005 \quad a_i^A = -0.0142 \quad a_i^A = 0.0388 \quad a_i^A = -0.0237 \]

\[ a_i^A = -0.5121 \quad a_i^A = 0.0477 \quad a_i^A = -0.2289 \quad a_i^A = 0.0399 \]

\[ a_i^A = 0.2287 \quad a_i^A = -0.0456 \quad a_i^A = 0.2175 \quad a_i^A = -0.0233 \]

\[ a_i^A = -0.0494 \quad a_i^A = 0.0171 \quad a_i^A = -0.0794 \quad a_i^A = 0.0055 \]

\[ a_i^A = 0.0042 \quad a_i^A = -0.0022 \quad a_i^A = 0.0101 \quad a_i^A = -0.0005 \]

2. Case B

\[ \beta_i = 0.5004 \quad \beta_i = 0.0594 \quad \beta_i = 0.3106 \quad \beta_i = -0.0355 \]

\[ \beta_i = -0.3810 \quad \beta_i = -0.2340 \quad \beta_i = -1.1127 \quad \beta_i = 0.1487 \]

\[ \beta_i = 0.02225 \quad \beta_i = -0.1120 \quad \beta_i = 1.9487 \quad \beta_i = -0.5206 \]

\[ \beta_i = 0.0728 \quad \beta_i = 0.1014 \quad \beta_i = -1.6151 \quad \beta_i = 0.5725 \]

\[ \beta_i = -0.0290 \quad \beta_i = -0.0781 \quad \beta_i = 0.5876 \quad \beta_i = -0.2353 \]

\[ \beta_i = 0.00348 \quad \beta_i = 0.01333 \quad \beta_i = -0.0762 \quad \beta_i = 0.03243 \]
3. Case C

\[ \tau_x = 0.9840 \quad \tau_y = 0.1228 \quad \tau_z = -0.28306 \]

\[ \sigma_x = 0.9970 \quad \sigma_y = -0.95545 \quad \sigma_z = 0.4371 \quad \sigma_i = -0.1098 \]

\[ \sigma_x = 0.0153 \quad \sigma_y = -0.0011 \quad \sigma_i = 0.00003 \]

4. Case D

\[ \delta^e = 0.5025 \quad \delta^i = 0.00233 \quad \delta^i = -0.0049 \quad \delta^i = 0.00237 \]

(Manuscript Received January 19, 1993, Accepted April 29, 1993)

旋窯焚化爐內部各表面間之擴散視因子

· 陳 宇 仰
私立元智工學院化工系

· 張 華 興
工研院能源與資源研究所

· 李 篤 中
國立臺灣大學化工系

摘要

一個旋窯焚化爐內部各表面間的熱傳遞對其效能有很大的影響。當要估算旋窯內部各不組表面間的輻射熱傳量時，需要先知道旋窯壁、固體床及旋窯兩端耐熱等各表面間的擴散視因子，但這些資料在文獻中仍未報導。在本文中，我們使用邊界積分及數值積分計算這些視因子，並將所得結果歸納成數個簡單型式的工作函數，以供未來進一步之應用。

Abstract

Thermal radiation heat transfer between enclosed surfaces in a rotary kiln incinerator is believed to play an important role in determining the system performance but has not been fully investigated. The diffuse view factors between these surfaces are the basic information for estimating the radiation heat flux but are still lack in literature. In this work, the diffuse view factors between the refractory wall, the solid bed, and the side walls are evaluated by contour integration follows by numerical integration. The results are correlated into several working equations which are simple in form and can be applied in further analysis.