

# HEAT AND MASS TRANSFER IN A ROTARY KILN INCINERATOR

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## ABSTRACT

A mathematical model of rotary kiln incinerator was developed. The model performs both heat and mass balances and heat transfer analyses in a rotary kiln combustor. The heat transfer mechanism includes radiative, convective and conductive heat transfer among combustion gases, the kiln wall and the solid bed. The model was used to predict the fuel requirement, flue gas properties, and axial temperature profiles of the solid bed, the kiln inner wall and outer skin. The model can be a useful tool in the design and operation of rotary kiln incineration system.

## INTRODUCTION

Years of phenomenal economical growth and wanton neglect has left the once pristine island, Taiwan, besieged with foul air, polluted water, contaminated soil and other equally distasteful environmental ills. Of the challenges faced Taiwan today, none is greater than those posed by the need to preserve its natural environment. Taiwan must establish quickly its indigenous environmental technology industry. The development of incineration technology is one of the major efforts that ERL has embarked in this area in last few years.

In 1987, the Ministry of Economic Affairs (MOEA) was commissioned to construct and operate a research facility for the thermal treatment of hazardous waste. The purposes of this facility were, and are, to establish a indigenous incineration industry and to support regulatory development and technology assessment. The pilot-scale incineration technology research facility (ITRF) include a multi-chamber incineration system (MIS) and a rotary kiln incineration system (RKI) representing the most common hazardous waste incineration equipment in use today.

Rotary kiln incinerators are currently being used widely as hazardous waste treatment facility. The RKI system can accept both solid and liquid wastes, and can be used to decontaminate soils or inert sorbants. Their characteristics, advantages and disadvantages can be found elsewhere [Chang, 1990].

In order to design the ITRF, a computer aided design/operation package was developed. This report presented a computer model of rotary kiln incinerator which was developed and used for the process design of the ITRF. The model performs both heat and mass balances and heat transfer analyses in a rotary kiln combustor.

The model was used to predict the fuel requirement, flue gas properties, and axial temperature profiles of the solid bed, the kiln inner wall and outer skin.

## HEAT AND MASS TRANSFER MODEL

Rotary kiln incinerator is a long, cylindrical rotating furnace in which solids and slurry are heated by

combustion of an auxiliary fuel [Manson and Unger, 1979]. The axis of the kiln makes an angle with the horizontal. The feed is introduced at the upper end of the kiln and the hot ash and combustion product discharge at the lower end. A schematic of the model is shown in Fig. 1.

The heat transfer mechanism includes radiative, convective and conductive heat transfer among combustion gases, the kiln wall and the solid bed. A two-sink model was used to calculate the radiative heat transfer within the freeboard. The heat transfer between the solid bed and the covered inner wall was modeled as direct surface-to-surface contact and radiation between two infinite parallel plates [Hottel and Sarofim, 1967].

The solid bed was considered to be well-mixed at any cross section [Watkinson et al., 1978]. In a rotary kiln, the free flowing particles give a linear cascading surface to the radial direction, as shown in Fig. 2. The cascading angle  $\beta$ , corresponding to the dynamic angle of repose at a low speed of rotation, can be expressed as [Sugimoto, 1981]

$$\beta = C_1 N_c + C_2 \quad (0.1 \leq N_c \leq 0.3) \quad (1)$$

where  $N_c$  is the critical speed ratio,  $C_1$  and  $C_2$  are empirical constants. In order to simplify the mathematical manipulation, a flat surface was assumed in the analyses.

The combustion gas temperature was also considered uniform. The gas was taken to be radiatively gray in the calculation of gas emissivity. The thermal decomposition rate of the solid waste was modeled as a first order chemical reaction.

## Energy Transport in the Waste Bed

The waste flows from the upper entrance toward the lower exit end. The rotation of the kiln provides continuous mixing of the solids and continuously renewed contacts between solids and the hot walls as well as direct contact with the hot gases. Due to the well-mixing of solid bed caused by the rotation of kiln [Sugimoto, 1981], the temperature of the waste was assumed to be approximately uniform at each axial location.

Continuity equation for the waste bed can be

expressed as follows.

$$dm_z/dz = -\Gamma S k(T_{sw}) \quad (2)$$

Enthalpy change in the waste bed equals to the radiative heat flux minus the enthalpy flow due to the volatilizing of waste.

$$d[m_z H_z]/dz = l q_f + l_a q_w - \Gamma S k(T_w) H_v \quad (3)$$

where

$$l = 2 R \sin(\alpha/2) \quad (4)$$

$$l_a = \alpha R \quad (5)$$

$$S = R^2(\alpha - \sin \alpha)/2 \quad (6)$$

First order chemical reaction was assumed in most waste incineration analyses. The rate constant  $k(T_w)$  can be expressed as

$$k(T_w) = A \exp[-E/R_a T] \quad (7)$$

The temperature in the waste bed was considered uniform, and is a function of only the axial coordinate,  $z$ .

$$T_{sw} = T_z \quad (8)$$

The enthalpies expressed in terms of specific heats were as follows:

$$H_z = C_p (T_z - T_0) \quad (9)$$

$$H_v = C_p (T_z - T_0) + E \quad (10)$$

The residence time of solid particles in a rotary kiln is a function of  $L/D$ , slope of inclination, and rotation speed [Perry and Chilton, 1989]. Therefore, the cross sectional area of waste bed can be related to the mass flow rate as follows.

$$S = m_z \Omega / \Gamma L \quad (11)$$

where

$$\Omega = 0.19 (L/D)/(\tau N) \quad (12)$$

The surface area of waste and refractory surface varies axially because of the burning of the waste.

### Thermal Radiation between Flame and Refractory Surface

Flame in a rotary kiln can be classified as cylindrical flame generated by auxiliary fuel burner and surface flame on waste.

The burner flame was assumed to be cylindrical in shape and to have a constant diameter and length. The combustion rate of fuel is specified as a function of distance from the burner. It was assumed that the flow have a uniform temperature inside the flame zone.

The surface flame was assumed to consist of a flat plane located at the top of the waste and is assumed to have a uniform temperature.

Radiation heat transfer between flame and refractory surface was calculated using Gebhart's absorption factor method [Gebhart, 1971]. Gebhart defines an absorption factor,  $B_{ij}$ , as the fraction of the emission rate of surface element  $a_i$ , which is absorbed by  $a_j$ , taking into account all paths whereby this radiant energy may

reach  $a_j$ . The absorption factors are a function of shape factors,  $F_{ij}$ , and the emissivity,  $\epsilon_i$ , of the surfaces.

The rate of radiant energy loss from a surface element  $a_j$  was calculated by,

$$q_j = \epsilon_j \sigma T_j^4 - \sum B_{ij} \epsilon_i \sigma (a_i/a_j) T_i^4 \quad (13)$$

where  $B_{ij}$  are solutions of the following simultaneous equations:

$$B_{ij} = F_{ij} \epsilon_j + \sum F_{ik} r_k B_{kj} ; j = \text{const.} \quad (14)$$

For the surface element  $a_j$ , equation (14) can be transposed and rearranged to obtain

$$\sum (F_{ik} r_k - \delta_{ik}) B_{kj} + F_{ij} \epsilon_j = 0 ; j \quad (15)$$

Equation (15) can be solved numerically for the  $N$  unknowns  $B_{kj}$  ( $j = \text{constant}$ ) by, such as, Newton-Raphson method.

### Heat Transfer from Surface Flame to the Waste

The heat transfer from surface flame to the waste  $q_f$  can be modeled as a radiative heat transfer between two infinite plates, and was given by

$$q_f = \sigma (T_f^4 - T_{sw}^4) / [1/\epsilon_f + 1/\epsilon_{sw} - 1] \quad (16)$$

### Heat Transfer from Inner Wall to the Waste

The heat transfer from inner refractory wall to the waste is primarily by surface-to-surface contact and radiation. The surface-to-surface contact heat transfer can be modeled by a convective-like way. The radiative heat transfer can be modeled as heat transfer between two infinite plates. Therefore, the total heat transfer rate is

$$q_w = \sigma (T_w^4 - T_{sw}^4) / [1/\epsilon_s + 1/\epsilon_{sw} - 1] + h_c (T_s - T_{sw}) \quad (17)$$

### Conductive Heat Transfer in the Refractory Wall

Since the refractory wall of the kiln are alternatively heated by the high temperature gas and cooled by solid waste charge during each revolution, unsteady heat transfer is present in the refractory wall.

For a coordinate system that is fixed to the rotating kiln, the temperature varies with both time and position. However, for a stationary coordinate system, the temperature remains constant. In order to simplify the mathematical manipulation, the stationary coordinate system was selected. In this case, the kiln wall has a velocity with respect to the coordinate system and the conductive heat transfer can be modeled as follows.

$$U \delta T / \delta \theta = \alpha_s (\delta^2 T / \delta x^2 + \delta^2 T / \delta z^2) \quad (18)$$

with the following boundary conditions:

$$-k_s \delta T / \delta r = -q_w ; 0 \leq \theta \leq \alpha \quad (19)$$

$$-k_s \delta T / \delta r = q_j ; \alpha \leq \theta \leq \pi \quad (20)$$

where the fill angle  $\alpha$  varies with  $z$  due to the combustion of waste.  $\alpha$  can be solved from equations (6), (11), and (12).

### METHOD OF SOLUTION

The resulting set of nonlinear differential/algebraic

equations are solved numerically by finite difference scheme. A computer program was developed to solve the problem numerically. A separate program was developed for the solution of view factor and hence the absorption factors.

Input data required for the computer program are the thermal properties of the waste, thermal properties of the refractory, waste feed rate, dimension of the kiln, rotation speed of the kiln, slope of the kiln, refractory thickness, auxiliary fuel feed rate and the dimension of the burner flame. Guesses for all unknown temperatures and radiative properties are also required to initialize the computation procedure. The converged solution consists of wall temperature distribution, solid waste temperature, solid waste flow rate, and temperature of combustion gas product over the entire length of the kiln.

Important parameters used in the model predictions include heat transfer coefficients for convection from the hot gas to the waste and walls, and emissivities. The emissivities of the solid waste and inner wall were estimated at 0.8 and 0.75 respectively. The emissivity of flame and combustion gases were estimated at 0.3 and 0.15 respectively.

## RESULTS AND DISCUSSIONS

Typical model simulation results for the incineration of low heating value waste were shown in figures 3 and 4 for cocurrent and countercurrent flow cases respectively. For the incineration of low heating value waste, such as contaminated soil, auxiliary fuel are required to maintain the high temperature requirement in the kiln. From the model results, it can be found a high temperature zone in the entrance region existed in cocurrent flow case. However, in the countercurrent case, high temperature zone are located near exit. Therefore, countercurrent design is a better configuration for the incineration facilities that is used for site remediation purposes.

Figures 5 and 6 show the typical results for high heating value waste, the waste rubber. In most centralized waste treatment facility, the waste was blended to have a heating value as high as possible in order to minimize auxiliary fuel consumption. In this case, auxiliary fuel burner was not required. From the model results, it can be found that the highest temperature of both gas and solids are located at the exit of the combustion zone. Although burners are not required for the cases of treating high-heating-value waste, it is still required for the startup of the system.

## CONCLUSIONS

A mathematical model of rotary kiln incinerator was developed. The model performs both heat and mass balances and heat transfer analyses in a rotary kiln combustor. The heat transfer mechanism includes radiative, convective and conductive heat transfer among combustion gases, the kiln wall and the solid bed. The model was used to predict the fuel requirement, flue gas properties, and axial temperature profiles of the solid bed, the kiln inner wall and outer skin. The model can be a useful tool in the design and operation of rotary kiln incineration system.

## Nomenclature

$A$	: Arrhenius constant
$E$	: Activation energy
$F_{ij}$	: View factor
$H^0$	: Enthalpy
$H_v$	: Enthalpy of volatilized gas
$h_c$	: Direct contact heat transfer coefficient
$m_c$	: Solid mass flow rate
$N_c^*$	: Critical speed ratio
$q_f^c$	: Heat transfer from surface flame to waste
$q_w^c$	: Heat transfer from inner wall to waste
$R$	: Radius of rotary kiln
$R_a$	: Gas constant
$S^a$	: Cross section area of solid bed
$T$	: Temperature of the solid refractory wall
$T_s$	: Temperature of solid waste
$T^{sw}$	: Reference temperature
$U^0$	: Circumferential velocity of refractory wall
$\alpha$	: Fill angle
$\alpha_s$	: Thermal diffusivity of refractory
$\beta^s$	: Cascading angle
$\tau$	: Slope of kiln
$\Omega$	: Residence time
$\epsilon$	: Emissivity of gas
$\epsilon_s^g$	: Emissivity of solid wall
$\epsilon_s^s$	: Emissivity of solid waste
$\Gamma^{sw}$	: Bulk density of waste bed

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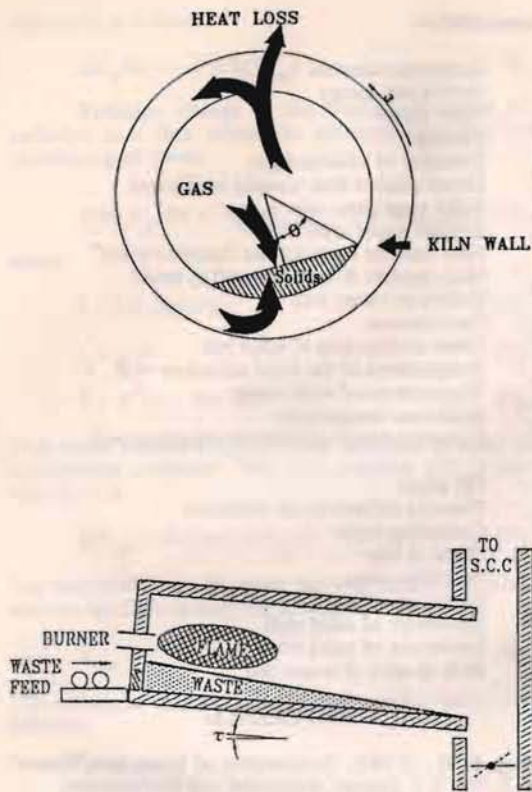


Fig. 1 Schematic diagram of a rotary kiln for heat transfer modeling.

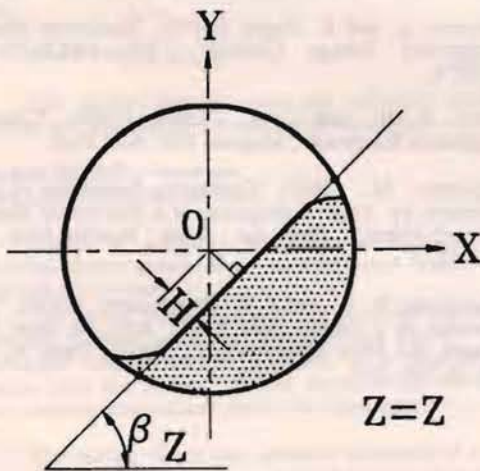


Fig. 2 Typical cross section of particle bed in a rotary kiln.

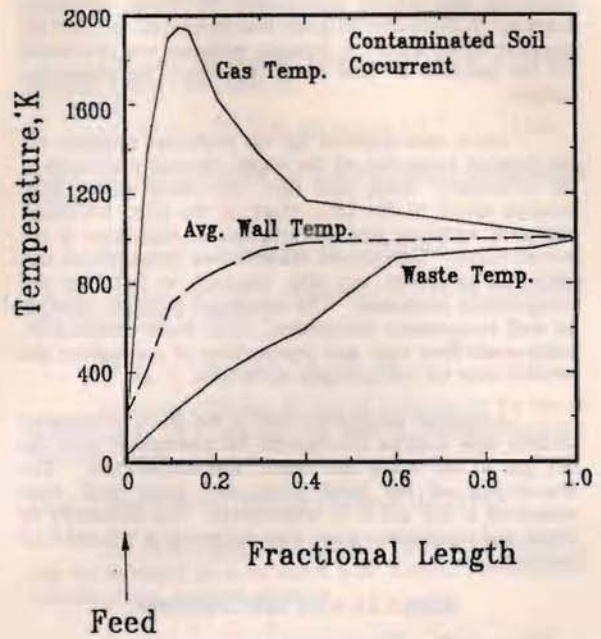


Fig. 3 Model predictions of gas, waste, and wall temperature profiles for cocurrent flow of gas and waste. (Contaminated soil)

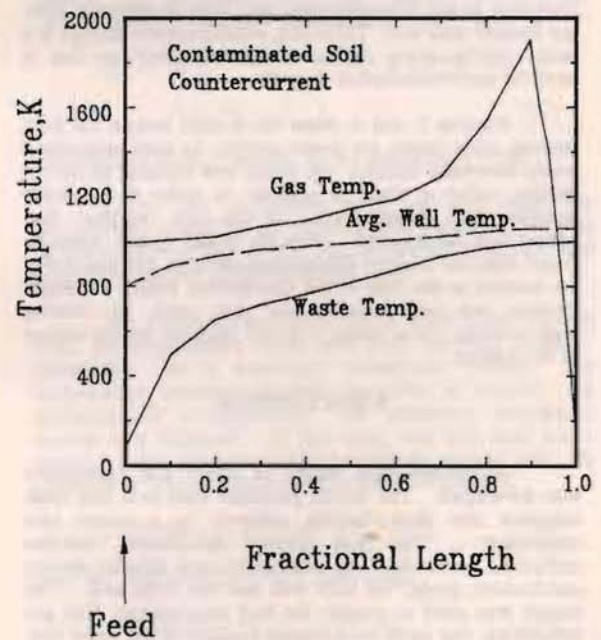


Fig. 4 Model predictions of gas, waste, and wall temperature profiles for countercurrent flow of gas and waste. (Contaminated soil)

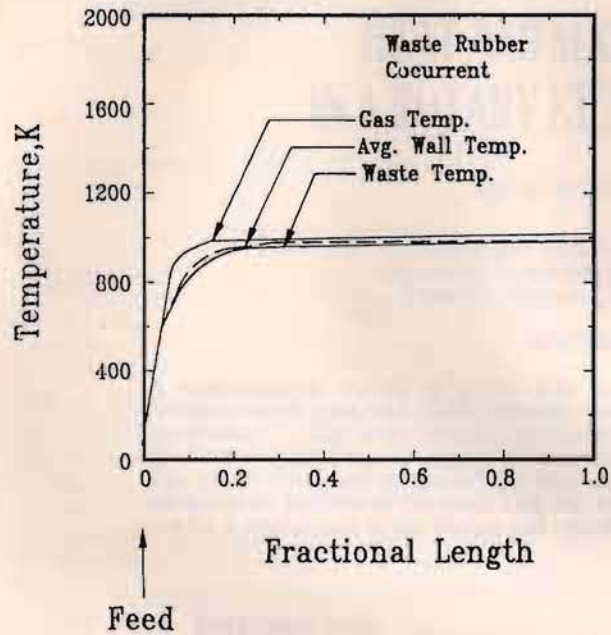


Fig. 5 Model predictions of gas, waste, and wall temperature profiles for cocurrent flow of gas and waste. (waste rubber)

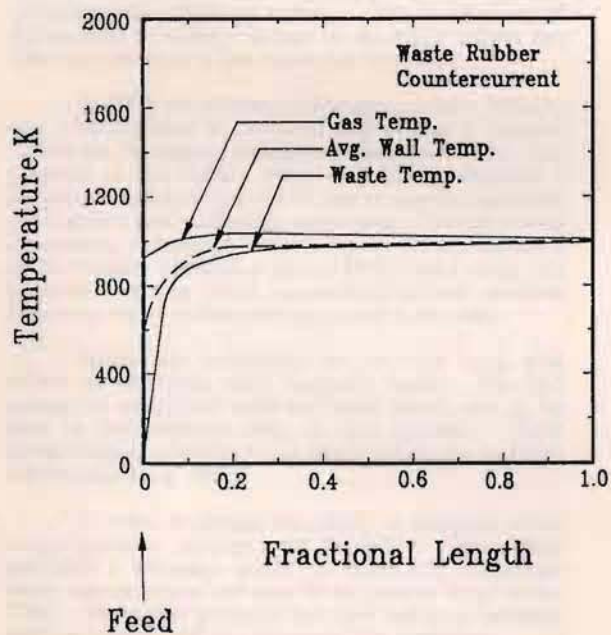


Fig. 6 Model predictions of gas, waste, and wall temperature profiles for countercurrent flow of gas and waste. (waste rubber)