Scale-up Design of A Multiple Impeller Gas-Liquid Contactors

After a success in a laboratory study and passing a favorable economic evaluation, the process will be subjected to a scale up stage. The objective of the scaling process is to obtain the process result can be as good as the one obtained in the laboratory test. In general, a small vessel has much intensive mixing which leads to an ideal uniformity and higher transfer rates. Once the vessel size enlarges to a plant scale, it is difficult to keep this ideal environment. Therefore, to have a successful scale up job, the engineer should grasp and understand the comprehensive information of the process and know what differences and examine why these differences occur between the laboratory scale unit and the plant scale equipment.

It is quite often to perform a scale-up process by applying some similarity principles, they are geometrical similarity, dynamic similarity, kinematic similarity and chemical or process similarity. Geometrical similarity is assumed that the ratios of major dimension between model and prototype systems to be a constant and it is more easily to follow than to design a dissimilar one. However this approach quite often may not give the same process performance as geometrically dissimilar system could make in plant scale production. Therefore, it is seldom to adopt geometric similarity alone in scale up a reactor unless the scaling ratio is small.

Dynamic similarity is to keep the ratios of the major related forces, such as inertia force, viscous force, gravitational force, and surface tension etc., to be a constant while the kinematic similarity asks all the flow characteristics to be the between the model and prototype systems. Since in scaling up a process, it limits the process fluid should be the same, thus it is rather difficult to keep both dynamic and kinematic similarity to accomplish a scale up process. Oldshue(1983) presented a table as shown in Table 9-1 which illustrates this argument. As shown in this table, to keep the value of Reynold’s number the same, it will result unreasonably low speed of the impeller, and to have the same circulation rate it requires an extreme large power requirement for plant scale unit. To maintain the tip velocity of impeller to be constant, it results a low average shear distribution and a low transfer rates for
the plant scale unit.

Table 9-1 Scale up of A Mechanically Agitated G-L Contactor.

<table>
<thead>
<tr>
<th>Operation Variables</th>
<th>Pilot Scale (5 gal.)</th>
<th>Plant Scale (625 gal.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>1.0</td>
<td>3125</td>
</tr>
<tr>
<td>P/V</td>
<td>1.0</td>
<td>25</td>
</tr>
<tr>
<td>N</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>D</td>
<td>1.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Q</td>
<td>1.0</td>
<td>125</td>
</tr>
<tr>
<td>Q/V</td>
<td>1.0</td>
<td>0.34</td>
</tr>
<tr>
<td>V_tip</td>
<td>1.0</td>
<td>1.7</td>
</tr>
<tr>
<td>Re.</td>
<td>1.0</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Although, there are various scale up criteria often used scaling up process of gas-liquid contacting systems, such as:

(1) Geometrical similarity,
(2) Energy dissipation density,
(3) Mixing time;
(4) Circulation time or pumping capacity,
(5) Transfer rates,
(6) Shear rate,
(7) Flow pattern of fluid
(8) Agitation intensity.

However, it is well recognized that to stick on any single scale up criterion is not practical for the most cases. Westerterp(1963) indicated that keeping D/T ratio and equal tip velocity, it will give a equal interfacial area for a gas-liquid contacting systems. Oldshue(1983) and Bourne(1964) suggested that scale-up a gas-liquid contacting process, to adopt energy dissipation density could provide a good scale up result. Moo-Young and Blanch(1981), and Nishikawa et al.(1981) pointed out to keep \( k_{L}a \) and constant gassing flowrate per liquid volume will give the best process result in plant scale vessels.

In this chapter, attention is devoted to interpret available information and research results on design and scale-up of multiple impeller agitated gas-liquid contactors. At first dimensions and design variable related to such system will be described to provide a basis for further discussion. Secondly, general principles of scale up of a multiple impeller gas-liquid system will be discussed along with the special case of scale up of a glutamic acid fermenter will be presented. At the end of this chapter, examples of process calculation based on different bases to determine operation variables will be described to facilitate the reader how to apply the
9.1 Design Variables

Fig. 9.1-1 shows multiple impeller agitated gas-liquid contactors with turbine impellers and with combination of turbine and pitched blade impellers. The major dimension and commonly used nomenclatures are also shown.

Besides these major dimensions, other variables which are very important to scale-up or design are fluid properties, impeller size, the ratio of liquid depth to tank diameter, H/T, power drawn by each impeller, and impeller tip velocity etc.

**Energy Dissipation Density** or P/V: Power consumed in the gas-liquid contactor is not only the principal cost of the mixing operation, but also reflects the rates of transfer properties, degree of gas dispersion and liquid circulation rate as well.

For a given value of K_a, there normally exists a minimum P/V which can be used as a basis for scale up process. In a multiple impeller gas-liquid contactor, power drawn by each impeller is not uniform unless the impeller speed is high enough. Designer should properly select type and combination of impellers to have not only a better gas dispersion but also to have more uniform shear and transfer property distributions. In turbulent flow regime, the power drawn by impeller is proportional to fluid density around the impeller, to the fifth power of impeller diameter and the third power of its rotational speed. Table 9.1-1 lists ranges of power consumption per unit volume of liquid in various processes.
Table 9.1-1 The ranges of energy dissipation density adopted for various processes.

<table>
<thead>
<tr>
<th>Type of Operation</th>
<th>Ranges of Energy</th>
<th>Dissipation Density</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KW/m³</td>
<td>HP/m³</td>
</tr>
<tr>
<td>Dissolution</td>
<td>0.15-1.50</td>
<td>0.20-2.00</td>
</tr>
<tr>
<td>Gas Adsorption</td>
<td>0.38-1.50</td>
<td>0.50-2.00</td>
</tr>
<tr>
<td>Heat Transfer</td>
<td>0.35-1.12</td>
<td>0.50-1.50</td>
</tr>
<tr>
<td>1-l Mixing</td>
<td>0.75-1.50</td>
<td>1.00-2.00</td>
</tr>
<tr>
<td>Suspension of Solids</td>
<td>1.12-2.25</td>
<td>1.50-3.00</td>
</tr>
<tr>
<td>Fermentation</td>
<td>0.75-3.00</td>
<td>1.00-4.00</td>
</tr>
</tbody>
</table>

**D/T ratio:** the ratio generally ranges from 0.2 to 0.5 for the most mechanically agitated gas-liquid contacting systems. The higher rotational speed is needed for small D/T ratio which may cause higher operation cost. The large D/T ratio may cause high tip velocity and low average shear for the system. Oldshue (1983) pointed out that as the ratio is 1/3, will give an optimum interfacial shear ratio for the system.

**H/T ratio:** For small volume system, this ratio = 1 is mostly adopted. For large volume system, high aspect ratio and multiple impeller system are preferred. Because it will give a better gas utilization and higher A/V ratio will provides a larger heat transfer area for the system. The ratio exceeding 3 is not recommended, since the liquid depth may exerts an effect on biological cells in the system and longer shaft also causes eccentric motion of the shaft. Multiple impeller system is generally recommended for high aspect ratio vessel to provide more uniform gas dispersion and shear distribution. Proper design of baffle plate is needed for such system to assure a better axial mixing within the system. To accelerate axial mixing or to reduce power consumption of the system, a combination of axial flow impeller with turbine impeller is often a popular selection.

Table 9.1-2 Variation of A/V with the Increase of Vessel Size.

<table>
<thead>
<tr>
<th>Size</th>
<th>D(m)</th>
<th>H/D=1</th>
<th>70% Full</th>
<th>A/V</th>
<th>H/T=1</th>
<th>70%</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 l</td>
<td>0.14</td>
<td>2.16x10⁻³</td>
<td>7.7x10⁻²</td>
<td>3x10⁻³</td>
<td>10.1x10⁻²</td>
<td>35.6</td>
</tr>
<tr>
<td>500 l</td>
<td>0.75</td>
<td>0.135</td>
<td>2.04</td>
<td>0.35</td>
<td>2.28</td>
<td>6.48</td>
</tr>
<tr>
<td>50 m³</td>
<td>3.10</td>
<td>21.0</td>
<td>17.4</td>
<td>35.0</td>
<td>25.2</td>
<td>0.83</td>
</tr>
<tr>
<td>100 m³</td>
<td>4.20</td>
<td>53.0</td>
<td>62.5</td>
<td>70.0</td>
<td>78.6</td>
<td>1.12</td>
</tr>
</tbody>
</table>

**Heat Transfer Area:** A G-L reaction system in a mechanically agitated vessel is characterized as a volume type system which amount of reaction heat is proportional to reaction liquid volume, however the wall area available for heat transfer, A, is proportional to (length)² while the volume space for reaction liquid is proportional to (length)³, thus the ratio of (wall area available)/(volume of liquid) for removing this heat to maintain system temperature will sharply decrease as the volume of the system increases. Table9.1-2 describes
this trend. Therefore, the designer should consider insertion of internal heat transfer element, such as cooling coil, or baffle coil etc., to maintain system temperature as specified.

**Liquid side mass transfer film coefficient, \( K_{L,A} \)**

Beside adopting energy dissipation density, \( P/V \), the value of liquid side mass transfer film coefficient is also very common to be used as a scale up basis. Laboratory test normally will give a suitable value of \( K_{L,A} \) and the pilot plant test can provide an optimum \( P/V \) value for the given value of \( K_{L,A} \). It is worthy to notice that the increase in viscosity of liquid will severely reduce the value of the mass transfer coefficient.

**9.2 Scale-up process for a mechanically agitated gas-liquid contactor**

As it was mentioned in the previous section, gas-liquid reaction system in a mechanically agitated vessel is characterized as a volume system, therefore, in scaling process, it is rare to stick on complete geometrical similarity and often adopt to examine applicability of combining both geometrical similar and a constant energy dissipation energy. Table 9.2-1 lists how the dimensions of the system would vary under these conditions and Table 9.2-2 gives the similar trend for which keeps both geometrical similarity and liquid circulation rate constant. Beside adopting energy dissipation density, \( P/V \), the value of liquid side mass transfer film coefficient is also very common to be used as a scale up basis, for gas-liquid contacting system, by keeping \( ND^{2/3} \) as constant can provide \( k_{L,A} \) as constant under geometrical similarity.

Moo-Young and Blanc (1981) pointed out a similar value of \( K_{L,A} \) could be achieved by Dissimilar geometric condition such as increasing the aspect ratio for plant scale unit. Table 9.2-3 illustrates their argument.

**Table 9.2-1 Ratio of Dimensions under Geometrical Similarity and \( P/V=\text{Constant} \).**

<table>
<thead>
<tr>
<th>( V_r )</th>
<th>( T )</th>
<th>( A_C )</th>
<th>( D )</th>
<th>( N )</th>
<th>( V_{\text{tip}} )</th>
<th>( Q )</th>
<th>( Q/V )</th>
<th>( H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.26</td>
<td>1.59</td>
<td>1.26</td>
<td>0.86</td>
<td>1.08</td>
<td>1.72</td>
<td>0.86</td>
<td>1.17</td>
</tr>
<tr>
<td>5</td>
<td>1.71</td>
<td>2.93</td>
<td>1.71</td>
<td>0.70</td>
<td>1.20</td>
<td>3.49</td>
<td>0.70</td>
<td>1.43</td>
</tr>
<tr>
<td>10</td>
<td>2.15</td>
<td>4.65</td>
<td>2.15</td>
<td>0.60</td>
<td>1.29</td>
<td>5.97</td>
<td>0.60</td>
<td>1.67</td>
</tr>
</tbody>
</table>

**Table 9.2-2 Ratio of Dimensions under Geometrical Similarity and \( Q/V=\text{Constant} \).**

<table>
<thead>
<tr>
<th>( V_r )</th>
<th>( T )</th>
<th>( D )</th>
<th>( N )</th>
<th>( V_{\text{tip}} )</th>
<th>( P/V )</th>
<th>( Q )</th>
<th>( Q/V )</th>
<th>( H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.26</td>
<td>1.26</td>
<td>1</td>
<td>1.26</td>
<td>1.59</td>
<td>2</td>
<td>1</td>
<td>1.59</td>
</tr>
<tr>
<td>5</td>
<td>1.71</td>
<td>1.71</td>
<td>1</td>
<td>1.71</td>
<td>2.93</td>
<td>5</td>
<td>1</td>
<td>2.93</td>
</tr>
<tr>
<td>10</td>
<td>2.15</td>
<td>2.15</td>
<td>1</td>
<td>2.15</td>
<td>4.65</td>
<td>10</td>
<td>1</td>
<td>4.65</td>
</tr>
</tbody>
</table>
Table 9.2-3 Variation of operation variables as keeping $K_La$ as constant.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Lab. Unit</th>
<th>Plant Scale Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.08 m³</td>
<td>10.0 m³</td>
</tr>
<tr>
<td>$H/T$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$P/V$</td>
<td>1</td>
<td>1 &gt;1</td>
</tr>
<tr>
<td>VVM</td>
<td>1</td>
<td>1 0.2</td>
</tr>
<tr>
<td>$V_s$</td>
<td>0.1</td>
<td>0.5* 0.1</td>
</tr>
<tr>
<td>$K_La$</td>
<td>1</td>
<td>- 1</td>
</tr>
</tbody>
</table>

* under flooding condition

It is also recognized that the upper limit to maintain a complete geometric similar in vessel dimension is approximately 10 m³, beyond this size, the system with single stage impeller is very hard to provide ideal uniform distribution of both gas hold-up and transfer properties. Since most commercial gas-liquid contactors usually has volume exceeds 10 m³ or diameter exceeds 1m, higher aspect ratio vessel is often preferred. In designing such a large scale mechanically agitated gas-liquid contactors, one should consider:

1. Appropriate type of impeller to have good gas dispersion
2. Adoption of multiple impeller system to up-grading gas utilization and to have uniform bubble and shear distributions.
3. Sufficient power input to have good macro mixing and uniform micro mixing and high mass transfer rates.
4. Having enough heat transfer surface to remove or supply heat of the process.
5. Proper design of baffle plate to have well mixing in axial direction.

### 9.3 Scale up of a gas-liquid contactor based on degree of mixedness

Although energy dissipation density is very common basis in scaling process, but it may not realistic for all cases (Paul and Treybal, 1971, Lu et al. 1979). Furthermore, almost all correlation proposed by previous researchers were obtained from stirred tanks with standard configuration which has only single impeller. Fig. 9.3-1 gives a plot process result vs. value of $P/V$ which the data were obtained from a pilot scale 0.5 m³ fermenter to examine with the possibility of $P/V$ as a scale up basis. In smaller $P/V$ region or low impeller speed region, the system with single impeller gives better process result than the system with dual impellers. Because in the region of small $P/V$, the dual impeller system can not disperse gas effectively and also will construct two segregated circulating flow regions which harms the homogeneity of the system. However these disadvantages will soon disappear as the value of $P/V$ increases to a certain degree. As shown in Fig. 9.3-1 when the value of $P/V$ exceeds approximately 1.36 Hp/m³, the performance of the dual impeller system becomes better than the single impeller system and reaches a level off point of 1.36 Hp/m³ while the single impeller system would not
reaches its level off point until $P/V$ reaches to a point of $2.4 \text{Hp}/\text{m}^3$. The result shown here implies $P/V$ may not be appropriate basis for scaling process for volume type processes such as fermenter with high aspect ratio. The similar result was also reported by Paul and Trebal (1971).

**Fig. 9.3-1 Plot of process result vs. energy dissipation density.**

In this section, how the quality of mixing affects the performance of fermenters as an example to illustrate how degree of mixing can be used as scaling up basis. To express the quality of mixing, many authors have used the mixing time or the circulation time, however since different criterion (0.1% to 5%) have been adopted by different authors to determine the end point of the tracer test, it is almost impossible to use the published correlations to predict mixing time for design purpose. To describe the quality of mixing, a population balance model proposed by Curl(1963) and modified by Evaglista, Katze and Shinnar(1969) is applied to describe the mixing phenomenon in a stirred vessel. In a stirred vessel, it can be assumed that

1. Reaction mass is made up of a large number of equally-size parcels material.
2. These parcels are undergoing independent pair collisions, equalizing concentrations, then separate
3. The system is considered as macro-complete mixing

Then a population balance for a given concentration over a system can be written as (Curl, 1963)

$$
\frac{\partial p(c, \theta)}{\partial c} = \frac{1}{\tau} [p_0(c, \theta) - p(c, \theta)] + 2\beta \sum p(c', \theta) P(c'', \theta)
$$

$$
\sum \delta(\frac{c' + c'' - c}{2}) \Delta c' \Delta c'' - p(c, \theta) + K \frac{\partial [c^2 P(c, \theta)]}{\partial c}
$$

where $\tau$ is the mean retention time of fluid, and $\beta$ is the ratio of the number of collision to the total number of parcels in the system, which can be considered as the mixing intensity
To interpret the parameter, $\beta$, a tracer mixing experiment can be designed for a batch mixing system without chemical reaction. Fig. 9.3-2 shows a typical time course of the tracer experiment. Thus Eq.(9.3-1) can be related to

$$\frac{\partial p(c, \theta)}{\partial c} = 2\beta \cdot \{ \int [p(c', \theta)P(c'', \theta) \cdot \delta(\frac{c'+c''}{2} - c)dc'dc'' - p(c, \theta)] \} \quad (9.3-2)$$

Introducing the definition of the mean or equilibrium concentration of the system

$$\bar{\mu}(\theta) = \int cp(c, \theta)dc = \text{const} \tan t \quad (9.3-3)$$

and the variance of the concentration distribution for a given $\theta$

$$\sigma^2(\theta) = \int (c - \bar{\mu})^2 p(c, \theta)dc \quad (9.3-4)$$

and multiplying $(c - \mu)^2$ to both sides of Eq.9.3-2, it gives

$$\frac{\partial[(c - \mu)^2p(c, \theta)dc]}{\partial c} = 2\beta \cdot \{ \int (c - \mu)^2 p(c', \theta)P(c'', \theta) \cdot \delta(\frac{c'+c''}{2} - c)dc'dc'' - 2\beta(c - \bar{\mu})^2 p(c, \theta)dc \} \quad (9.3-5)$$

which can be reduced to

$$\frac{\partial \sigma^2(\theta)}{\partial \theta} = \beta \sigma^2(\theta) \quad (9.3-6)$$

Integrating Eq.9.3-6 gives

$$\sigma^2(\theta) = \sigma^2(0)e^{-\beta \theta} \quad (9.3-7)$$

Fig.9.3-2 Typical time course of tracer concentration of a tracer experiment.

This result implies that in a batch tracer mixing experiment, the nonhomogeneity in the distribution of the tracer in the system decays simply with the time constant $\beta$. This equation also suggested that the value of the mixing intensity parameter, $\beta$, can be obtained from the
magnitude of the slope in the plot of $\ln \sigma^2(\theta)$ vs. $\theta$. Fig. 9.3-3 shows a typical plot of $\sigma^2(\theta)$ vs. $\theta$, while Fig. 9.3-4 shows the plot of mixing intensity $\beta$ vs. $P_v/V$ and $\beta$ vs. $P_g/VD^2$ respectively. A straight line obtained in the plot of $\beta$ vs. $P_g/VD^2$ in Fig. 9.3-4 supports a theoretical equation derived by Corrsin (1957) for a turbulent mixer.

\[
\beta = \frac{21.6}{A^2 \eta} \left( \frac{P_g}{V D^2} \right)^{1/3}
\]  

(9.3-8)

where $A^2 \eta$ is independent of the scale factor and can be determined by the physical properties and configuration of the system.

To see how the mixing intensity can be used as a scale-up criterion, the data converted from Fig. 9.3-1 through the equations described here are plotted against $\beta$, as shown in Fig. 9.3-5. A unique relation between process yields and $\beta$ can be obtained whether a single or dual impellers is used. This single relationship between process result and operational
parameters is more useful to fin a scale up criterion than result shown in Fig. 9.3-1.

![Fig.9.3-5 The use of $\beta$ as a basis of scale up process.](image)

**9.4 Relationship between $\beta$ and Average Fluctuation Velocity**

Under the assumption of locally isotropic and homogeneous turbulent field in a tank, a Fourier transformation and order of magnitude analysis of Navier-Stokes equations will give a relation between agitation power per unit mass, $\varepsilon$, and turbulent intensity.

$$\varepsilon = \frac{\eta p}{\rho V} = 20.625 \frac{u'^3}{D}$$  \hspace{1cm} (9.4-1)

where $\eta$ is the efficiency of power input. Furthermore, Schwarzberg has presented a relationship between local velocity fluctuation and tank variables as

$$u' = B [ND^2 / (T^2 H)^{1/3}]$$  \hspace{1cm} (9.4-2)

where $T$ is the characteristic length, and tank diameter. As the Reynolds number of the system is larger than $10^4$, for a completely baffled, straight-blade turbine impeller, the value of its power number, $N_p$, approaches a constant value of 6. Substituting this relationship and $V = (\pi / 4)T^2$ into Eq.(9.4-1), one has an expression relating the root mean square of fluctuation velocity $u'$ and the operation variables as

$$u' = (1.54h)^{1/3} \frac{ND^2}{(T^2 H)^{1/3}}$$  \hspace{1cm} (9.4-3)

Eq.(9.4-2) and (9.4-3) can be used in scaling up stirred tanks if $u'$ can be served as a basis scale-up of agitated tanks. Applying the collision frequency $\beta$, Evangelister gave

$$\beta = \frac{6V}{\lambda^2}$$  \hspace{1cm} (9.4-4)

where is kinematic viscosity of fluid and $\lambda$ is Corrsin’s energy dissipation scale. For an isotropic homogeneous turbulent field, Corrsin has derived an equation relating $\beta$ and $u'$ as
Combining Eq.(9.4-3),(9.4-4) and (9.4-5), it yields

\[ \beta = 37.125 \frac{u'}{T} \]  

(9.4-6)

As seen in Eq.(9.4-6), there exists a linear relationship between the intensity of agitation, \( \beta \) and the turbulent intensity. Since, \( \beta \), represents the number or frequency of collisions of liquid particles per unit volume per unit time, it can be interpreted as an overall expression for the mixing intensity of a batch system. However, to determine the value of \( \beta \) for a given system requires a laborious tracer experiment. On the other hand, \( u' \) represents the intensity of turbulence of an isotropic turbulent field, and it can be measured more simply by using a hot film anemometer or laser anemometer. Isotropic turbulent assumption might not be applied to a large agitated tank; it is reasonable to use the overall average of \( u' \) of various location within the system to express the intensity of mixing of the agitation system.

In Fig.9.4-1, the data of \( \beta \) obtained from tracer experiments are plotted against the corresponding value of \( <u'> \). A straight line shown in the plot verifies the linear relationship given to Eq.9.4-6. This will be a great improvement if the volume average fluctuating velocity of the system can be used as a scale-up criterion in designing agitated tanks, since the procedure to determine the collision coefficient for a given system requires laborious tracer experiments, while fluctuating velocity can be measured more easily using a hot film or a laser Doppler anemometer.

![Fig. 9.4-1 The linear relationship between \( \beta \) and \( <u'> \).](image-url)
9.5 Effect of Average Fluctuating Velocity on Rate of Gas-Liquid Reactions

To demonstrate how the fluctuating velocity within the system would affect the rate of reaction, the oxidation of sodium sulfite was performed batchwise in a 30L tank without buffering the PH value. The conversion with time was measuring by sampling at certain time intervals. Prior to evaluating the rate constant, the reaction order was first determined for each data set by the kinetic method. The rate constant and initial reaction rate were then calculated. Figs 9.5-1 and 9.5-2 show the initial reaction rates vs. $<u'>$ for the catalyzed and non-catalyzed oxidation of sodium sulfite, respectively. As $<u'>$ is less than 0.1 m/sec, the data disperse irregularly in both figures and no correlation can be assigned. The mixing in this range falls into the category of poor dispersion.

Fig. 9.5-1 The effect of $<u'>$ on initial rate of a catalyzed reaction.

Fig. 9.5-2 The effect of $<u'>$ on initial rate of a non-catalyzed reaction.
The scatter of the data may be caused by the unstable turbulent mixing pattern. As $<u'>$ is larger than 0.1 in/sec, linear relationships are obtained for both cases, reaction rate is proportional to $<u'>$^{1.25} for catalytic reactions while it is proportional to $<u'>$^{1.7} for non-catalytic reactions. These implies that the reaction rates of oxidation of sulfite are dominated by the volume average fluctuating velocity, and $<u'>$ can be used as scale-up basis for stirred tank system. (Lu and Chen, 1986)

9.6 Example of Scale-up of Glutamic Acid Fermenters Bases on Mixing Intensity

During year of 1959, the design of this first 50 m³ fermenter as shown in Fig.9.6-1 was adopted for hasty production of glutamic acid was after the design of fermenter used for penicillin fermentation. Neither mass transfer rate nor energy dissipation density, except cooling area was considered.

![Fig.9.6-1 50 m³ fermenter.](image1)

![Fig.9.6-2 100 m³ Fermenter with helical coil.](image2)

Surprisingly, this fermenter has had only 10% lower process result than that of 500 liter pilot plant scale fermenter. Therefore after completion of erection and operation of six the same 50 m³ fermenters, a 100 m³ fermenter as shown in Fig.9.6-2 was erected. For the first 100 m³ fermenter, helical coils which has larger heat transfer coefficient than the snake type coil which was used in 50 m³ fermenters was adopted for cooling propose. The average process result obtained from this new design fermenter is slightly lower than that of 50 m³ tanks, but it was still in acceptable range. However, as the culture medium was changed from crude glucose solution to diluted molasses solution, several defective phenomena appeared; (1)
15-20% lower yield. (2) difficult to control the system temperature. The engineer in charge
production discarded the helical coil and inserted a large numbers of snake shape coil in the
fermenter as illustrated in Fig. 9.6-3.

The improper insertion of these coil created poor mixing for this fermenters. Fig. 9.6-4
compares the mixing time in these fermenters vs. the impeller rotational speed with the results
of Kiya(1970). This comparison clearly indicates a severely poor mixing quality in these plant
scale fermenters. This poor mixing could be due to

(1) improper insertion of the snake too many coils,
(2) improper selection of the type of impellers.

![Fig. 9.6-3 100 m³ Fermenter with snake type coils.

To examine the effect of mixing on fermentation process, a series of experiments were
conducted and the results were illustrated as shown in Fig.9.3-1and Fig. 9.4-5. To improve the
design of the existed 100 m³ fermenter, firstly, the 25% of coils at each coner were removed to
improve the cross flow of liquid between outer and inner cores for the fermenter. Secondly,
the two 45° pitch paddle impellers were replaced by standard Rushton turbine impeller and the
plots of $\beta$ against the yield and the $\beta$ vs. $P_v$ is used to re-design type and size of impellers
such an optimum power input can be obtained. Table 9.6-1 lists a comparison of the fermentation on month average performances before and after the improvement. From the values of yield and product concentration shown in this table, it clearly indicates that the quality of mixing affects considerably the yield and productivity in glutamic acid fermentation.

**Table 9.6-1 Performance of 100 m$^3$ fermenter before and after modification.**

<table>
<thead>
<tr>
<th></th>
<th>Before Modification</th>
<th>After Modification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield</td>
<td>42-43%</td>
<td>50-52.7%</td>
</tr>
<tr>
<td>Final GA Concentration</td>
<td>5.5-6.3%</td>
<td>7.6-7.7%</td>
</tr>
<tr>
<td>Rating among 16 Fermenters</td>
<td>15$^{th}$ and 16$^{th}$</td>
<td>2$^{nd}$ and 4$^{th}$</td>
</tr>
</tbody>
</table>

![Fig. 9.6-5 300 m$^3$ fermenter.](image)

Basing on the similar data of $\beta$, a new 300 m$^3$ fermenter was designed as shown in Fig.
9.6-5, the performance (yield and final concentration etc.) was as well as expected.

9.7 Design of a Multiple Impeller Gas-Liquid Contactor

Following examples described here are to illustrate how the correlations which were discussed in this monograph can be applied to estimate the required value of design variables for a plant scale multiple impeller gas-liquid contactor which has working liquid volume of 300 m$^3$. Three commonly adopted criteria, i.e. the energy dissipation density $P_g/V$ (Section 9.5.1), overall volumetric average mass transfer coefficient $<K_{a}>$ or $<K_{G,a}>$ (Section 9.5.2) and average turbulent intensity $<u'>$ (Section 9.5.3) are shown here with the requirements and given condition of the design. In all of these examples, the geometrical configurations, including the impeller type, D/T ratio and the clearance between impellers, have been presumed and are identical for all examples. Figure 9.7-1 shows the interaction between the correlations obtained in this study, from which one can estimate the rotational speed of the impeller for the required multiple impeller gas-liquid contactor to have the expected performances of the system.

**Fig. 9.7-1** The flow diagram for estimating the rotational speed of the impeller to suit each requirement and given conditions.

9.7.1 Design Based on the Energy Dissipation Density $P_g/V$
Design Basis

From pilot tests, the best condition to produce xxx aminoacid in a aerobic fermenter was found to be $P_g/V$ is 1.4 hp/m$^3$ of broth under gassing rate of 0.5 VVM or 2.5 m$^3$/sec.

Geometrical Dimension of the Plant Scale Fermenter:

A triple Rushton impeller layout is recommended for such a high aspect ratio system, the height of static liquid level $H=7/3T$, and the diameter of the vessel $D=T/3$ are set while the distance between each impeller will apart $C=2D$ is also adopted. Since the required working liquid volume is $V=300$ m$^3$, from $V=\pi/4T^2H$, it gives the tank diameter $T=5.47$m and impeller diameter $D=1.82$m. From the reference table, the semi elliptical bottom for $D=5.47$ m it will give 11.5 m$^3$ volume and a height of 1.06 m, thus $H$ (the real liquid height) = $H'$ + radius of the semi-elliptical bottom. The height static level for cylindrical part can be estimated by $(V_{total}-V_g)=\pi/4T^2H'$, which gives $H'=12.6$m and $H=H'+H_e=12.6+5.4\times0.194=13.7$m. Fig. 9.7-2 gives a sketch of geometrical dimensions of the proposed contactor.

Fig. 9.7-2 Dimensions of the proposed contactor design.

*Calculated examples presented in 9.7-1 were contributed by Dr. H. H. Wu.
Calculation procedure

Fig. 9.7-3 illustrates the calculation procedure to estimate the required operation condition based on a given value of energy dissipation energy or $P_g/V$.

For the first trial, assuming $N=150$ rpm = 2.5 rps and checking the aeration number, $N_A(=Q_s/ND^3)$. It results:

$$N_A = Q_s/ND^3 = 2.5[(150/60)(1.82)^3] = 0.166 > 0.03.$$
1. Estimation of the net sparged gas rate for the middle and upper impellers

\[ \frac{Q_{Sn}}{Q_{Si}} = 3.2N^{0.28}Q_{Si}^{0.23}n_g^{-0.474}(D/T)^{1.69} \quad n_g > 1 \]

\[ Q_{S2} = 0.875 \text{m}^3/\text{sec} \]

\[ Q_{S3} = 0.722 \text{m}^3/\text{sec} \]

2. Estimation of the gas recirculation rate around each impeller

\[ \frac{Q_{Rn}}{Q_{Sn}} = 0.043N^{1.05}Q_{Sn}^{-1.12}(D/T)^{-1.15} \quad N_A = 0.166 > 0.03. \]

\[ Q_{R1} = 0.357 \text{m}^3/\text{sec} \]

\[ Q_{R2} = 0.405 \text{m}^3/\text{sec} \]

\[ Q_{R3} = 0.414 \text{m}^3/\text{sec} \]

3. Estimation of the total gassing rate around each impeller

\[ Q_{t1} = Q_{S1} + Q_{R1} = 2.857 \text{m}^3/\text{sec} \]

\[ Q_{t2} = Q_{S2} + Q_{R2} = 1.280 \text{m}^3/\text{sec} \]

\[ Q_{t3} = Q_{S3} + Q_{R3} = 1.136 \text{m}^3/\text{sec} \]

4. Estimation of the power drawn by each impeller

Since \((1-P_g/PO) = 4.69N_A + 0.032 = 4.69(Q_{Sn}/ND_3) + 0.032\) for \(Q_{Sn}/ND_3 > 0.03\)

The ungassed power drawn by a single Rushton turbine impeller, \(P_O\), was evaluated by assuming \(N_p = PO/\rho N^3D^5 = 4.5\), therefore \(P_O = 1404.072 \text{W}\)

\((1-P_g/PO)_1 = 0.921\) or \(P_{g1} = 110,922 \text{W}\)

\((1-P_g/PO)_2 = 0.430\) or \(P_{g2} = 800,321 \text{W}\)

\((1-P_g/PO)_3 = 0.386\) or \(P_{g3} = 862,100 \text{W}\)

5. Total power consumption under a gassed condition

\(P_{g, total} = P_{g1} + P_{g2} + P_{g3} = 1773,343 \text{W}\)

i.e. \(P_{g, total}/V = 5911.1 \text{W/m}^3 = 7.88.45 \text{hp/m}^3 >> 1.4 \text{hp/m}^3\)

6. Using trial and error approach and, repeat the above procedure until the \(P_{g, total}/V\) criterion was matched. If \(N=90\text{rpm}\), the calculated results shows that

\(Q_{S2} = 1.01 \text{m}^3/\text{sec}\) and \(Q_{S3} = 0.833 \text{m}^3/\text{sec}\)

\(Q_{R1} = 0.209 \text{m}^3/\text{sec}\) \(Q_{t1} = Q_{S1} + Q_{R1} = 2.709 \text{m}^3/\text{sec}\)

\(Q_{R2} = 0.232 \text{m}^3/\text{sec}\) \(Q_{t2} = Q_{S2} + Q_{R2} = 1.242 \text{m}^3/\text{sec}\)

\(Q_{R3} = 0.238 \text{m}^3/\text{sec}\) \(Q_{t3} = Q_{S3} + Q_{R3} = 1.071 \text{m}^3/\text{sec}\)

\((1-P_g/PO)_1 = 0.951\) or \(P_{g1} = 14,861 \text{W}\)

\((1-P_g/PO)_2 = 0.676\) or \(P_{g2} = 98,263 \text{W}\)

\((1-P_g/PO)_3 = 0.588\) or \(P_{g3} = 124,951 \text{W}\)

where \(P_O = 303,280 \text{W}\)

\(P_{g, total} = P_{g1} + P_{g2} + P_{g3} = 238,075 \text{W}\)
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i.e. \( \frac{P_{g,\text{total}}}{V} = 793.6 \text{W/m}^3 = 1.06 \text{hp/m}^3 < 1.4 \text{hp/m}^3 \)

In final trial, we assume \( N = 102 \text{rpm} = 1.7 \text{rps} \), and \( \frac{P_{g,\text{total}}}{V} \) converged at 1.4hp/m\(^3\). Therefore, 102 rpm should be the final rotational speed of the shaft to have energy dissipation density (\( \frac{P_{g,\text{total}}}{V} \)) of 1.4hp/m\(^3\). The system equipped with a hybrid of the Rushton turbine impeller and pitched blade impeller can be calculated follow the similar procedure, while the correlations in Chap 6 for the pitched blade impeller should be applied to estimate the power drawn by the pitched blade impeller in this flow diagram.

9.7.2 Design Based on the Mass Transfer Coefficient \( <K_La> \) or \( <K_Ga>P \)

Given Conditions

Average mass transfer coefficient \( <K_La> = 0.043 \text{ l/s} \) and gassing rate \( Q_S = 0.5 \text{ VVM} \)

**Dimensions of the Plant Scale Fermenter:**

the same as the example given in Fig. 9.7-2.

**Design Requirements:**

1. Scale up basis: keep the same \( <K_Ga>P \) with laboratory test (P=total gas phase pressure)
   \( (K_GaP)_2 = (K_GaP)_1 = 0.01391 \text{ mol/m}^3\text{s} \)

2. Tank pressure at liquid free surface \( P = 1.2 \text{ atm} \)

   (a) Mass transfer rate

   Since \( \frac{1}{K_Ga} = \frac{H}{K_La} \approx \frac{H}{K_La} \),

   \[ K_Ga = \frac{K_La}{H} \approx \frac{0.043}{0.649} = 0.6626 \frac{\text{mole}}{\text{m}^3\cdot\text{s}\cdot\text{atm}} \]

   \( N_1a \approx K_GaP_{O_2} = (0.06626)(0.21) = 0.01391 \text{ mol/m}^3\text{s} \)

   (b) Oxygen Balance in Plant Scale Fermenter

   Since the basis of scale-up is given as: \( (K_GaP)_2 = (K_GaP)_1 = 0.01391 \text{ mol/m}^3\text{s} \), the pressure at the bottom of the plant scale fermenter \( P_{2_b} \) is:

   \[ P_{2_b} = P_{2_f} + 13.7/10.34 = 1.2 + 1.33 = 2.53 \text{ atm} \text{. If } T=15^\circ\text{C}=288\text{K}, \]

   Molar flow rate of air and inert nitrogen are:

   \[ G_{air} = 2.5 \times \frac{273}{288} \times \frac{2.53}{1.0} \times \frac{1}{22.4 \times 10^{-3}} = 267.7 \text{ mol/s} \]

   \[ G_{N_2} = 0.79 G_{air} = 267.7 \times 0.79 = 211.5 \text{ mol/s} \]

   Then, the net oxygen flow rate over the fermenter can be given as:
Solving the above equation for $P_{f,O_2}$ ($P_{O_2}$ at liquid free surface) gives:

$$P_{f,O_2} = \frac{0.245}{0.3699}$$

Thus,

$$P_{O_2} = P_{f,O_2} = 0.245\text{atm}$$

(c) Evaluation of the rotational speed

$$\therefore (K_Ga)_2 = \frac{N da}{P_2} = 0.01391/0.3699 = 0.0376 \frac{\text{mole}}{m^3 \cdot \text{s} \cdot \text{atm}}$$

$$(K_La)_2 = H \times (K_Ga)_2 = 0.0376 \times 0.649 = 0.02441/\text{s}$$

Take the overall average mass transfer coefficient $<K_La>_D = \frac{1}{n} \sum <K_La>_n$

The correlation for $<K_La>_D$ in multiple impeller system was given as:

$$<K_La>_D = 0.134(Q_{m}/ND^3) + 0.00339$$

where subscript $n$ is the impeller stage and $N$ denotes the rotational speed of the impeller in rps.

**Calculation procedure**

Start calculation with $N=110\text{rpm}=1.7\text{rps}$ initially,

1. Estimation of the net sparged gas rate for the middle and upper impellers

   $$Q_{S1} = 2.5\text{m}^3/\text{sec}$$
   $$Q_{Sn}/Q_{S1} = 3.2N^{0.23}Q_{S1}^{0.474}(D/T)^{1.69}$$
   $$n_s > 1$$
   $$Q_{S2} = 0.955\text{m}^3/\text{sec}$$
   $$Q_{S3} = 0.788\text{m}^3/\text{sec}$$

2. Estimation of the gas recirculation rate around each impeller

   $$Q_{Rd}/Q_{Sn} = 0.043N^{0.05}Q_{Sn}^{-1.12}(D/T)^{-1.15}$$
   $$N_e = 0.166 > 0.03.$$  
   $$Q_{R1} = 0.258\text{m}^3/\text{sec}$$
   $$Q_{R2} = 0.289\text{m}^3/\text{sec}$$
   $$Q_{R3} = 0.296\text{m}^3/\text{sec}$$

3. Estimation of the total gassing rate around each impeller

   $$Q_d = Q_{S1} + Q_{R1} = 2.758\text{m}^3/\text{sec}$$
   $$Q_{d2} = Q_{S2} + Q_{R2} = 1.244\text{m}^3/\text{sec}$$
   $$Q_{d3} = Q_{S3} + Q_{R3} = 1.084\text{m}^3/\text{sec}$$

4. Estimation of the $<K_La>_D$ around each impeller
Fig. 9.7-4 The flow diagram for estimating the rotational speed of the impeller based on a given $<K_{L}a>$ value.
\[ \langle K_L \alpha \rangle_D = 0.134 \left( \frac{Q_m}{ND^3} \right) + 0.00339 \]

\[ \langle K_L \alpha \rangle_{D1} = 0.0368 \text{ l/s} \quad \langle K_L \alpha \rangle_{D2} = 0.0185 \text{ l/s} \quad \langle K_L \alpha \rangle_{D3} = 0.0165 \text{ l/s} \]

(5) Comparison of the calculated \( \langle K_L \alpha \rangle_{ave} \) value with the given \( \langle K_L \alpha \rangle \)

\[ \langle K_L \alpha \rangle_{ave} = \frac{1}{3} [\langle K_L \alpha \rangle_{D1} + \langle K_L \alpha \rangle_{D2} + \langle K_L \alpha \rangle_{D3}] \]

\[ \langle K_L \alpha \rangle_{ave} = 0.0239 \text{ l/s} < 0.0244 \text{ l/s} \]

Try again and show that when \( N = 105 \text{ rpm} \), \( \langle K_L \alpha \rangle_{ave} \approx 0.244 \text{ l/s} \) \[ \Rightarrow N = 105 \text{ rpm} = 1.75 \text{ rps} \]

Fig. 9.7-4 shows the flow diagram for estimating the operating rotational speed of the triple Rushton turbine impeller system based on \( K_L \alpha \).

**9.7.3 Design Based on the turbulent intensity \( u' \)**

**Given Conditions**

\( Q_{Si} = 0.5 \text{vvm} = 150 \text{m}^3/\text{min} = 2.5 \text{m}^3/\text{sec} \) and average turbulent intensity \( u' = 4.53 \text{ m/s} \)

**Dimensions of the Plant Scale Fermenter:** the same as the example given in the section 9.7-2.

**Design Requirements:**

1. What is the rotational speed of impeller should be if the scale up basis is \( u' = 4.53 \text{ m/s} \).

**Evaluation of the rotational speed**

1. From the results of Cutter (1996), the value of \( u' \) could be related to \( K_L \alpha \) and \( Q_s \) as:

\[ \varepsilon = \frac{P_g}{V} = 20.62u'/D \]

Substitution of \( u' = 4.53 \text{ m/s} \) into the above equation, it gives \( \frac{P_g}{V} = 1053.2 \text{ W/m}^3 = 1.4 \text{ hp} \).

Follow the same procedure described in Fig. 9.7-5, one can obtain the rotational speed \( N \) as \( 102 \text{ rpm} = 1.7 \text{ rps} \# \)

For a multiple impeller system with a hybrid of the Rushton turbine impeller and pitched blade impeller can be designed based on the procedure similar to those described above, except the correlations for the pitched blade impeller should be added into the calculation. Table 9.7-1 shows the comparison of the calculated results of these three design examples based on different criteria. From the data shown in this table, it can be found that the results of all these design cases agree with each other very well, and the design basis can be chosen from any one of these three criteria.
Fig. 9.7-5 The flow diagram for estimating the rotational speed of the impeller based on a given $<u'>$ value.
Table 9.7-1 Comparison of the calculated values of N based on different design criteria.

<table>
<thead>
<tr>
<th>Related characteristics</th>
<th>Design basis</th>
<th>$P_g/V$ (W/m$^3$)</th>
<th>$&lt;K_{L,a}&gt;$ (1/sec)</th>
<th>$&lt;u'&gt;$ (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy dissipation density</td>
<td>$P_g/V$ (W/m$^3$)</td>
<td>1.4hp/m$^3$</td>
<td>1.42hp/m$^3$</td>
<td>1.4hp/m$^3$</td>
</tr>
<tr>
<td></td>
<td>1050W/m$^3$</td>
<td>1065W/m$^3$</td>
<td>1050W/m$^3$</td>
<td></td>
</tr>
<tr>
<td>Mass transfer coefficient</td>
<td>$&lt;K_{L,a}&gt;$ (1/sec)</td>
<td>0.0238</td>
<td>0.0244</td>
<td>0.0238</td>
</tr>
<tr>
<td>Turbulent indensity</td>
<td>$&lt;u'&gt;$ (m/sec)</td>
<td>4.53</td>
<td>4.59</td>
<td>4.53</td>
</tr>
<tr>
<td>Rotational speed</td>
<td>N (rps)</td>
<td>1.7</td>
<td>1.75</td>
<td>1.7</td>
</tr>
<tr>
<td>Tip velocity $U_{tip}=\pi ND$</td>
<td>(m/sec)</td>
<td>9.72</td>
<td>10.00</td>
<td>9.72</td>
</tr>
</tbody>
</table>

**NOTATION**

- $a$ Interfacial area per unit of liquid volume [1/m]
- $a_{proj}$ Projected area in I direction [m$^2$]
- $B$ Baffle width [m]
- $c$ Tracer concentration [M]
- $C$ The clearance between impellers [m]
- $C_1$ The off-bottom distance of lowest impeller [m]
- $D$ Impeller diameter [m]
- $H$ Liquid height of stirred tank [m]
- $k_{L,a}$ Mass transfer coefficient [m/hr]
- $K_{L}$ Liquid film mass transfer coefficient [m/s]
- $K_{L,a}$ Mass transfer coefficient [1/s]
- $<K_{L,a}>$ Volume-averaged mass transfer coefficient [1/s]
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$\langle K_{i,a} \rangle_t$ Volume-averaged mass transfer coefficient around each impeller [1/s]

$L_{res}$ Resultant turbulence scale [m]

$M$ The mixing index defined by Ogawa(1981) [-]

$N$ Impeller rotational speed [1/s]

$N_b$ Number of bubbles [-]

$N_p$ Power number [-]

$n_b$ Number of baffles [-]

$n_s$ The stage of impeller (>1) [-]

$p(c, θ)$ Probability density function of concentration [-]

$\rho$ Pressure [kg/m$^3$/s$^2$

$P_0$ Ungassed impeller power consumption [kgm$^2$/s$^3$

$P_b$ Power consumption with baffle [kgm$^2$/s$^3$

$P_g$ Power consumption with aeration [kgm$^2$/s$^3$

$P_o$ Power consumption without aeration [kgm$^2$/s$^3$

$P_r$ Power consumption per unit volume [HP/m$^3$

$P_{gb}$ Power consumption without baffle [kgm$^2$/s$^3$

$Q_d$ Uprising gas rate along the take wall [m$^3$/s]

$Q_l$ Liquid phase flow rate between regions [m$^3$/s]

$Q_R$ Gas recirculation rate [m$^3$/s]

$Q_s$ Sparged gas flow rate [m$^3$/s]

$Q_t$ Total gassing rate (=Qs+QR) [m$^3$/s]

$q_i$ Liquid phase flow rate between cells for mixing time simulation [m$^3$/s]

$R$ Impeller radius [m]

$R_l$ A combination of operating variables (=N$^{3.15}D^{6.85}/\rho^{0.1}Q^{0.252}T^{2}H$) [m]

$S$ Overall interfacial area [m$^2$

$t$ Time [s]

$t_M$ Mixing time [s]

$T$ Tank diameter [m]

$u'$ The fluctuation velocity [m/s]

$U_{lg}$ Liquid velocity with aeration [m/s]

$U_{lu}$ Liquid velocity without aeration [m/s]

$U_{tp}$ Tip velocity of the impeller [m/s]

$V$ Liquid volume in the tank [m$^3$

$V_s$ Gas superficial velocity [m/s]

$vvm$ The ratio of aeration rate per minute /the liquid volume in the tank [-]

$W$ Impeller blade width [m]

$z*$ Dimensionless axial coordinate (=2z/w) [-]

<$\text{Greeks Letters}$>

$\alpha_i$ Local gas hold-up [-]

$\beta$ Mixing intensity [1/s]

$\epsilon$ Energy dispersion per unit mass of fluid [m$^2$/s$^3$

$\eta$ Efficiency of mixing power [-]

$\theta$ Time [s][hr]
\[ \theta \quad \text{Tangential coordinate} \quad [-] \\
\kappa \quad \text{Turbulent kinetic energy} \quad [m^2/s^2] \\
\mu \quad \text{Concentration} \quad [g/l] \\
\mu \quad \text{Viscosity} \quad [kg/m \cdot s] \\
\rho \quad \text{Density of fluid} \quad [kg/m^3] \\
\sigma^2 \quad \text{Variance of concentration distribution} \quad [g^2/l^2] \\
\sigma \quad \text{Deformation rate} \quad [1/s] \\
\tau \quad \text{Shear stress} \quad [kg/ms^2] \\
\]

\text{<Subscripts>}

\(<\quad \text{The average value} \\
0 \quad \text{Initial condition} \\
D \quad \text{At discharge region} \\
f \quad \text{Final condition} \\
g \quad \text{Gassed condition} \\
i \quad \text{Interface} \\
n \quad \text{The } n^{th} \text{ stage of impeller} \\
t \quad \text{total} \\
\)

\text{<Superscripts>}

\(* \quad \text{Saturated solubility of gaseous component at the gas-liquid interface} \\
- \quad \text{Average quantity} \\
\)

\text{<Dimensionless Groups>}

\[ F_r \quad \text{Frude number} \quad [N^2/D/g] \\
N_{Re} \quad \text{Reynolds number} \quad [\rho ND^2/\mu] \\
N_a \quad \text{The tradition aeration number} \quad [Q_g/ND^3] \\
N_a' \quad \text{The modified aeration number} \quad [Q_g/ND^3] \\
N_p \quad \text{Power number} \quad [P_g/\rho N^3D^5] \\
\]

\text{<Abbreviation>}

DR \quad \text{Deformation rate} \\
LDA \quad \text{Laser Doppler Anemometer} \\
TKE \quad \text{Turbulent kinetic energy} \\
PPP \quad \text{The system equipped with three pitched blade impellers} \\
PPR \quad \text{The system equipped with a lower Rushton turbine impeller and two upper pitched blade impellers} \\
PRR \quad \text{The system equipped with two lower Rushton turbine impeller and a upper Pitched blade impellers} \\
RRR \quad \text{The system equipped with three Rushton turbine impellers} \\